Calculating the Creep Life of Rotating Cylindrical Pressure Vessels by Reference Stress Method (RSM)

Farid Vakili-Tahami[1], Seyyed Saeed Sharifi[2], Peyman Majnoun[3], Afshin Abbasi[4]

[1] *Associate professor, Department of mechanical engineering, Tabriz University*

[2, 3, 4] *MSc. student, Department of mechanical engineering, Tabriz University*

Abstract

Due to the vast application of pressure vessels in different industrial fields such as gas and petrochemical plants and power generation, investigating their mechanical behavior is one of the major concerns in recent studies. These vessels usually operate at high temperature under internal pressure and axial rotation; therefore, their creep behavior should be studied. However, due to the nonlinear form of the creep constitutive equations, the numerical solution methods require sufficient and advanced software and hardware facilities. Hence, in most of the industrial standard codes such as R5, approximate procedures, which are based on reference stress method (RSM), are used. Consequently, the estimation of the reference stress is a priory for using these codes. Calculating reference stress in structures which are subjected to combined loadings is also a complicated process. For this purpose a method is proposed to obtain the reference stress value for vessels which are subjected to combined internal pressure and axial rotation in order to predict their creep life. Based on the Finite Element Model (Abaqus) the precision of the analytical solutions in determining the value of reference stress is verified by an acceptable maximum error of 10 percent. Then the optimum design of pressure vessels is performed by the purpose of maximizing the creep life and satisfying the strength criterion by using the reference stress. For instance a diagram of design parameters is prepared for a rotating cylindrical pressure vessel made of INCONEL 718 material at the operating temperature of 760 degree centigrade, which will be useful for designers.

Keywords: Optimum design, Creep, Reference Stress Method (RSM), Cylindrical Pressure Vessel, Creep Life

1. Introduction

Cylindrical tanks are one of the most widely-used components in various industrial and engineering systems. Pressure vessels and pipe lines in gas and oil industry, power generation, nuclear reactors, centrifuges, etc. can be pointed out. These tanks usually experience internal pressure and forces resulted from rotation in high temperatures. For this reason, stress and strain analysis resulted from creep in thick-walled cylindrical tanks is one of interesting topics for researchers. Sim and Penny (1971) analyzed a wide range of thick-walled tanks under creep conditions for various loading conditions including internal pressure, external loading, and inertia loading. Assuming plane strain conditions, Bhatnagar et al. (1984, 1986) analyzed homogeneous rotating cylinder under internal pressure facing steady-state creep. Also, they, in another study, studied creep analysis of thick-walled rotating cylinders considering the anisotropy effect on stress and strain (1986). Reference stress method explains non-elastic responses of structures. This method was developed to evaluate creep behavior of perfect components (1995, 1997). Another method to study the creep in tanks is numerical-analytical method in which it reaches stress and strain distribution in tank walls using governing-equations solved by numerical methods (2014). Sharma (2009) studied transient behavior of thick-walled rotating tanks using numerical-analytical methods. Pankij (2011, 2013, and 2014) forecasted elasto-plastic behavior of rotating tanks at constant temperature using limited deformation method. Also, Kashkoli and Zamani (2014) studied the effect of heat flux in creep behavior of thick-walled tanks with numerical-analytical method. Husseini et al. (2011) introduced accurate analytical method to determine creep behavior of rotating disks which experienced internal and external pressures. Although this method is widely used in combinational loads (2003, 2008), it is time taking and requires advanced hardware equipment. On the contrary, estimation methods, mainly based on reference stress calculation, are capable of predicting creep behavior of structures with short time solution. Therefore, this method is widely used in standard codes such as R5 (1991, 2011)

Determining reference stress in combinational loads is one of important challenges in approximation and estimation methods. For this reason, this paper intends to present a method to determine reference stress of cylindrical tanks which are under combinational loading resulted from internal and axial rotation. Hence, the results of this method are approved during a Finite Element Modeling. Then tank optimal design was performed according to reference stress in order to maximize creep life with meeting the strength criterion condition. The design parameter diagrams are presented for rotating cylindrical tanks made up of INCONEL 718 at 760 degrees Celsius which can be used by industrial designers.

2. STEADY CREEP ANALYSIS

The cylindrical tank under consideration is rotating. The specifications are as follows: *a* is internal radius, *b* is external radius, and the tank is under *p* internal pressure with ω rotating speed. It is assumed that plane strain and axially symmetric loading are chosen in cylindrical coordinate. The balance equation is as equation (1) in this coordinate:

$$r\frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r - \rho r^2 \omega^2 \qquad (۱)$$

Where *r* is the distance of each point on tank wall from the center, ρ is density, and σ is stress. r and θ sub-indexes are radius and perimeter directions. Equation (2) shows compatibility equation:

$$r\frac{d\dot{\varepsilon}_\theta^c}{dr} = \dot{\varepsilon}_r^c - \dot{\varepsilon}_\theta^c \qquad (۲)$$

Where ε strain and c upper script show creep. By principle of no change in volume of plasticity of equation (3), we can conclude that:

$$\dot{\varepsilon}_\theta^c + \dot{\varepsilon}_r^c + \dot{\varepsilon}_z^c = 0 \qquad (۳)$$

Where z sub index shows the longitudinal direction. According to plane strain of $\dot{\varepsilon}_z^c = 0$, then $\dot{\varepsilon}_\theta^c = -\dot{\varepsilon}_r^c$ Radius and perimeter creep strain rate is obtained by mixing this term in equation (2):

$$\frac{d\dot{\varepsilon}_r^c}{dr} = -\frac{2\dot{\varepsilon}_r^c}{r} \Rightarrow \dot{\varepsilon}_r^c = -\dot{\varepsilon}_\theta^c = \frac{C_1}{r^2} \qquad (۴)$$

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_r-\sigma_\theta)^2+(\sigma_r-\sigma_z)^2+(\sigma_\theta-\sigma_z)^2} \quad (۵)$$

$$\bar{\dot{\varepsilon}}^c = \sqrt{\frac{2}{3}}\sqrt{\left(\dot{\varepsilon}_r^c\right)^2+\left(\dot{\varepsilon}_\theta^c\right)^2+\left(\dot{\varepsilon}_z^c\right)^2} \quad (۶)$$

Replacing equation (4) in equation (6), effective creep strain rate is obtained:

$$\bar{\dot{\varepsilon}}^c = \frac{2}{\sqrt{3}}\frac{C_1}{r^2} \quad (۷)$$

According to plane strain and $\dot{\varepsilon}_z^c = \lambda S_{zz}$ Prandtl – Reuss law in which *s* is deviant stress, we can write:

$$\sigma_z = \frac{1}{2}(\sigma_r+\sigma_\theta) \quad (۸)$$

Replacing σ_z in equation (5), effective stress is as following:

$$\sigma_e = \frac{\sqrt{3}}{2}\left|\sigma_\theta-\sigma_r\right| \quad (۹)$$

Since steady creep analysis is desired, the relationship between effective creep strain rate and effective strain is expressed as Norton equation (1995):

$$\frac{d\,\bar{\varepsilon}^c}{dt} = B\sigma_e^m \quad (۱۰)$$

Where *B* and *m* are creep equation constants depending on the material. Mixing equation (1) and (9), we will have:

$$\sigma_e = \frac{\sqrt{3}}{2}\left[r\frac{d\sigma_r}{dr}+\rho\omega^2 r\right] \quad (۱۱)$$

Mixing equation (7) and (10), we will have:

$$\frac{2}{\sqrt{3}}\frac{C_1}{r^2} = B\sigma_e^m \quad (۱۲)$$

Mixing equation (11) and (12), we will have:

$$\frac{C_1}{r^2} = \left[\frac{\sqrt{3}}{2}(\mathrm{r}\frac{d\sigma_r}{dr})+\rho\omega^2 r\right]^m B \quad (۱۳)$$

Following integration and some mathematical calculations, we will have:

$$\sigma_r(r)=\frac{-m}{\sqrt{3}}\left(\frac{2C_1}{B\sqrt{3}}\right)^{\frac{1}{m}}r^{-\frac{2}{m}}-\frac{\rho r^2\omega^2}{2}+C_2 \qquad (۱۴)$$

Where C_2 is integration constant. Boundary conditions are as follows for stress:

$$\begin{cases}\sigma_r(r=a)=-p\\ \sigma_r(r=b)=0\end{cases} \qquad (۱۵)$$

We will reach C_1 and C_2 constants by imposing boundary conditions of (15):

$$\begin{cases}C_1=\left(B\frac{\sqrt{3}}{2}\right)\left[\dfrac{\sqrt{3}\frac{p}{m}+\frac{\sqrt{3}\rho\omega^2(b^2-a^2)}{2m}}{a^{-\frac{2}{m}}-b^{-\frac{2}{m}}}\right]^m\\ C_2=-P+\rho a^2\frac{\omega^2}{2}+\dfrac{a^{-\frac{2}{m}}\left(\rho\omega^2\left(\frac{a^2-b^2}{2}\right)-P\right)}{b^{-\frac{2}{m}}-a^{-\frac{2}{m}}}\end{cases} \qquad (۱۶)$$

Putting C_1 and C_2 in equation (14), we will obtain σ_r:

$$\sigma_r(r)=\frac{p}{(b/a)^{-\frac{2}{m}}-1}\left(\left(\frac{r}{a}\right)^{-\frac{2}{m}}-\left(\frac{b}{a}\right)^{-\frac{2}{m}}\right)+\frac{1}{2}\rho a^2\omega^2\left[1-\left(\frac{r}{a}\right)^2+\frac{(b/a)^2-1}{(b/a)^{-\frac{2}{m}}-1}\left(\left(\frac{r}{a}\right)^{-\frac{2}{m}}-1\right)\right] \qquad (۱۷)$$

Putting in equation (1), we will reach perimeter stress:

$$\sigma_\theta(r)=\frac{p}{(b/a)^{-\frac{2}{m}}-1}\left[\frac{m-2}{m}\left(\frac{r}{a}\right)^{-\frac{2}{m}}-\left(\frac{b}{a}\right)^{-\frac{2}{m}}\right]+\frac{1}{2}\rho a^2\omega^2\left[1-\left(\frac{r}{a}\right)^2+\frac{(b/a)^2-1}{(b/a)^{-\frac{2}{m}}-1}\left[\left(\frac{m-2}{m}\right)\left(\frac{r}{a}\right)^{-\frac{2}{m}}-1\right]\right] \qquad (۱۸)$$

Putting σ_r and σ_θ in equation (8), we will reach axial stress:

$$\sigma_z(r)=\frac{p}{(b/a)^{-\frac{2}{m}}-1}\left[\frac{m-1}{m}\left(\frac{r}{a}\right)^{-\frac{2}{m}}-\left(\frac{b}{a}\right)^{-\frac{2}{m}}\right]+\frac{1}{2}\rho a^2\omega^2\left[1-\left(\frac{r}{a}\right)^2+\frac{(b/a)^2-1}{(b/a)^{-\frac{2}{m}}-1}\left[\left(\frac{m-1}{m}\right)\left(\frac{r}{a}\right)^{-\frac{2}{m}}-1\right]\right] \qquad (۱۹)$$

Effective stress is obtained by putting equations (17) and (18) in equation (9):

$$\sigma_e(r)=\frac{p}{1-(b/a)^{-\frac{2}{m}}}\left(\frac{\sqrt{3}}{m}\left(\frac{r}{a}\right)^{-\frac{2}{m}}\right)+\frac{1}{2}\rho a^2\omega^2\left[\frac{(b/a)^2-1}{1-(b/a)^{-\frac{2}{m}}}\left(\left(\frac{\sqrt{3}}{m}\right)\left(\frac{r}{a}\right)^{-\frac{2}{m}}\right)\right] \qquad (۲۰)$$

Since obtained stress distribution is independent from *B* coefficient in structural equation of creep, dimensionless stress distribution needs to be determined for different *m* values in order to determine reference stress and point in the tank. Reference point is the point in which stress is equal for different *m*s (various materials). Therefore, dimensionless stress value is independent from tank material. This stress shows its importance. Structure behavior can be associated with experimental data from single axial creep test using this stress. For this reason, calculating reference stress is highly regarded (2011).

Whenever *m* takes a large value, the stress distribution will be constant at section by approaching it to infinity in *limit* condition. In this state, elastic is completely assumed plastic in which total tank section reaches plastic (yield) state after a steady creep at a certain pressure of angular velocity. This certain pressure of rotating speed is known as “plastic collapse”. P_L Pressure in equation (20) is obtained by assuming $\omega = 0$ and approaching *m* to infinity. Also, σ_Y is obtained by assuming effective stress equal with yield stress:

$$p_L = \frac{2\sigma_Y}{\sqrt{3}} Ln\left(\frac{b}{a}\right) \qquad (۲۱)$$

Also, ω_L is obtained in equation (20) by $p = 0$, approaching *m* to infinity and assuming equal effective stress and yield stress.

$$\omega_L = \sqrt{\frac{4\sigma_Y \, Ln\frac{b}{a}}{\sqrt{3}\rho a^2\left(\left(\frac{b}{a}\right)^2 - 1\right)}} \qquad (۲۲)$$

On the other hand, the relationship is expressed as follows between reference stress and plastic collapse (2011):

$$\sigma_R = \frac{p}{p_L}\sigma_Y \qquad (۲۳)$$

$$\sigma_R = \frac{\omega}{\omega_L}\sigma_Y \qquad (۲۴)$$

According to the meaning of equal reference stress for various materials (different *m*s), tank reference stress under internal pressure load and rotating speed is as follows, respectively:

$$\left.\sigma_R\right)_p = \frac{\sqrt{3}p}{2Ln\left(\frac{b}{a}\right)} \qquad (۲۵)$$

$$\left.\sigma_R\right)_\omega = \frac{\sqrt{3}\rho a^2\omega^2\left[\left(\frac{b}{a}\right)^2 - 1\right]}{4Ln\left(\frac{b}{a}\right)} \qquad (۲۶)$$

In state of loading with internal pressure or rotating speed, reference point is obtained by assuming correspondent reference stress in each state equal with equation (20) as follows:

$$r_R = \sqrt{\frac{2a^2Ln\left(\frac{b}{a}\right)}{1-\left(\frac{a}{b}\right)^2}} \qquad (۲۷)$$

In order to determine reference stress in combinational loading state, superposition principle can be inspired. This principle is valid in linear plastic loading, while creep behavior of structures is non-linear and plastic. For this reason, it is necessary to modify the equation inspired from superposition principle with α_i and β_i coefficients. This modification is performed by considering the desired accuracy and comparison of results. Therefore, the proposed equation is as follows:

$$\left(\sigma_R\right)_{com} = \sum_{i=1}^{q}\alpha_i\left(\sigma_{R,i}\right)^{\beta_i} \qquad (۲۸)$$

Where $\left(\sigma_R\right)_{com}$ is reference stress for combinational loading and $\sigma_{R,i}$ is reference stress for *i* independent loading. α_i and β_i are determined by comparing the results and desired accuracy. The reference stress is as follows for tank under internal pressure along with rotating velocity:

$$\sigma_R = \alpha_p\sigma_{R,p}^{\beta_p} + \alpha_\omega\sigma_{R,\omega}^{\beta_\omega} \qquad (۲۹)$$

$$\sigma_R = \alpha_p\left[\frac{\sqrt{3}p}{2(Ln\frac{b}{a})}\right]^{\beta_p} + \alpha_\omega\left[\frac{\sqrt{3}\rho a^2\omega^2\left[(\frac{b}{a})^2 - 1\right]}{4Ln\frac{b}{a}}\right]^{\beta_\omega} \qquad (۳۰)$$

Modification coefficient value is considered 1. The validity of this assumption is evaluated by comparing with results. If it is required, the values are modified. Therefore, reference stress is proposed as equation (31) for rotating cylinder under internal pressure:

$$\sigma_R = \frac{\sqrt{3}}{4} a^2 \left(\frac{\frac{2P}{a^2} + \rho\omega^2 (R^2 - 1)}{LnR} \right) \tag{۳۱}$$

Where R is external radius to internal radius of tank.

3. OPTIMUM DESIGN OF CREEP LIFETIME

Determining the tank thickness is highly regarded during optimum design of thick-walled cylindrical tanks in order to maximize creep lifetime at a certain temperature, pressure, and rotating speed. To this end, Optimum thickness can be reached by assuming constant internal tank radius and varying external radius, R, in order to minimize the reference stress for a certain internal pressure and rotating speed.

$$\frac{d\,\sigma_R}{dR} = \frac{\sqrt{3}}{4} a^2 \left[\frac{2R\rho\omega^2 \ln(R) - \frac{1}{R}\left(\frac{2P}{a^2} + \rho\omega^2 (R^2 - 1) \right)}{(LnR)^2} \right] = 0 \tag{۳۲}$$

So, we will have:

$$\frac{p}{\frac{1}{2}\rho a^2 \omega^2} = \left[1 + R_{opt}^2 \left(\ln\left(R_{opt}^2 \right) - 1 \right) \right] \tag{۳۳}$$

According to equation (33), there is only a single $R_{opt.}$ for a certain pressure and rotating speed which minimizes the reference stress. Creep lifetime will be maximum in this state. Fig. 1 shows ref. stress in three states including internal pressure loading, axial rotation, and combination for different radius ratios. This diagram was drawn for pressure of 30 Mpa and rotating speed of radians per second.

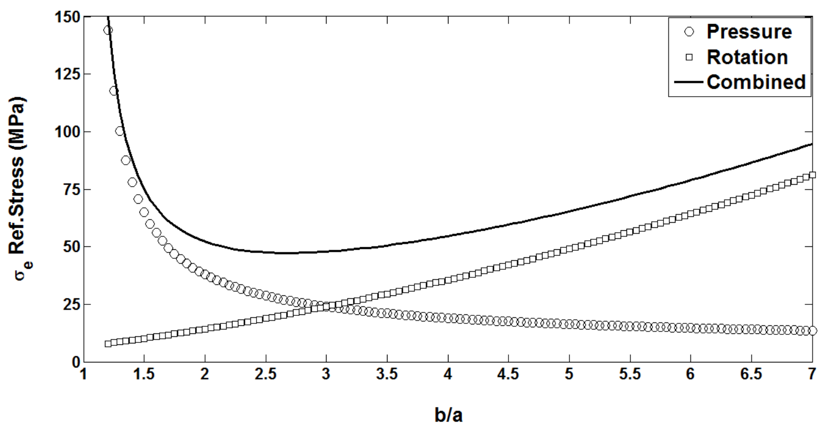


Fig. 1: Ref. stress changes for different radius ratios in internal pressure, axial rotation, and combined loading

According to Fig. 1, tank thickness rise leads to rotation share rise in ref. stress and accordingly, creep lifetime increase. Also, the effect of internal pressure is more obvious by thickness decrease. According to Fig. 1, in combinational loading state, minimum ref. stress is obtained at a certain pressure and speed only at a certain thickness. This result can be generalized for each certain pressure and speed so that an optimum thickness is introduced for ref. stress minimization and accordingly, maximum creep lifetime. This objective is satisfied with equation (33). Industrially speaking, tank specification needs to be determined in a way that it enjoys design creep lifetime. In addition to tank geometry, this issue depends on working temperature and its material. Equation (34) gives a certain pressure and rotating speed for each optimum thickness, providing creep lifetime from single axial test for S_R stress.

$$P \leq -\frac{1}{2}\left(R^2-1\right)\rho a^2\omega^2+\frac{2}{\sqrt{3}}Ln\left(R\right)S_R \tag{۳۴}$$

Larson-Miller diagram shows the relationship between temperature, stress, and creep lifetime. Using this diagram, we are able to determine creep lifetime for various temperatures (2005). Yet, it is necessary to point out that speed and pressure selection is also limited by strength criterion. Maximum Von- Mises stress in inner layer of tank in elastic state is as follows as strength criterion:

$$\sigma_e=\frac{\sqrt{3}R^2}{R^2-1}P+\frac{\sqrt{3}}{2}\rho a^2R^2\omega^2 \tag{۳۵}$$

We will have the following equation to meet Safety Factor (SF) in design.

$$P \leq -\frac{1}{2}\left(R^2 - 1\right)\rho a^2 \omega^2 + \frac{R^2 - 1}{\sqrt{3}R^2 SF}\sigma_Y \qquad (۳۶)$$

Where σ_Y is yield stress of material at working temperature.

In fact, an optimum thickness is introduced for a certain pressure and rotating speed which leads to minimum ref. stress in tank. Then creep lifetime can be predicted by strength collapse criterion.

4. NUMERICAL ANALYSIS

In this analysis, Finite Element Method was used for steady creep in cylindrical tank by the help of Abaqus software. Radial, axial, and perimeter stress distributions and finally effective stress in tank wall were obtained in three states: only internal pressure, only axial rotation, and combinational state of pressure and rotation using software output. Since Norton relationship power (m) is between 3 and 9 for most industrial materials (1980), effective stress distribution at tank wall is obtained $m = 1, 4, 9$ for Norton power coefficients according to numerical solution. Then ref. stress and ref. point were determined. In order to study the accuracy of numerical solutions, ref. stresses were compared with those of analytical solution. To reduce calculation time, tank was axially symmetrically modeled. Acceptable error was defined at $1*10^{-5}$ for strain. Tiny elements sized 0.001 of internal tank radius were used for grid. Also, according to convergence of responses, as many as 22000 elements were selected for each longitudinal section for numerical solution.

5. RESULTS

Dimensionless diagram of stress components is determined for different *m*s at tank wall and ref. stress and point are calculated in order to verify the results of numerical and analytical solutions. Based on mechanical specifications and dimensions of tank, complete plastic pressure and rotating speed are reported 253.7 Mpa and 901.8 radians per second, respectively. In order to meet appropriate safety factor, internal pressure and rotating speed were assumed $P = 0.25P_L$ and $\omega = 0.35\omega_L$

in numerical and analytical solution, respectively. Table 1 lists mechanical specification for INCONEL 718 at 760 degrees Celsius:

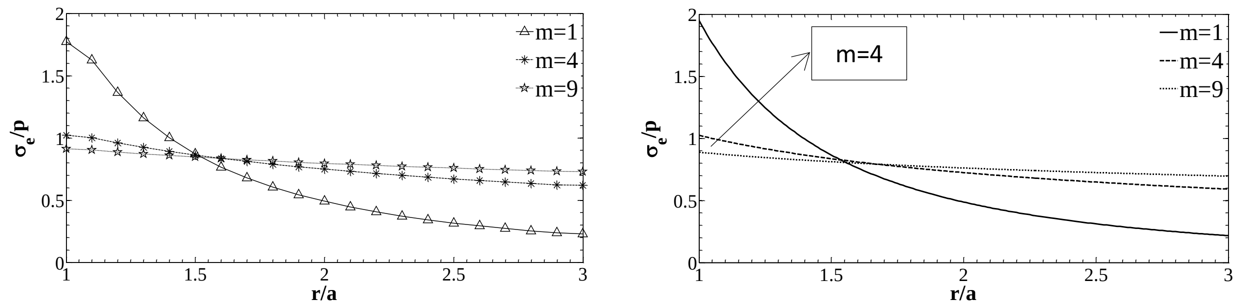
Table 1: Geometric dimensions and mechanical specifications of cylindrical tank (2014)

Internal radius	$a = 0.1\ [m]$
Radial ratio	$b/a = 3$
Yield stress	$\sigma_Y = 758\ [MPa]$
Elastic module	$E = 210\ [GPa]$
Poisson coefficient	$\nu = 0.3$
Density	$\rho = 7800\ [Kg/m^3]$
Stress for 1000-hour creep lifetime	$S_R = 172\ [MPa]$
Stress for 10000-hour creep lifetime	$S_R = 125\ [MPa]$

Fig. 2 shows effective dimensionless stress distribution at tank wall under 66-Mpa internal pressure. Fig. 2-a presents the results of analytical method and Fig. 2-b shows the results of Finite Element Analysis. Effective stress distribution for $m = 1$ is elastic stresses. Stress for other values of *m* shows the effective stress distribution for steady creep state. The effective stress has declined in internal surface of tank due to stress redistribution as a result of creep phenomenon in tank. In Fig. 2, the intersections of diagrams show ref. stress and point which are reported 56.1 and 52MPa, respectively.

Fig. 3 shows effective dimensionless stress distribution in tank wall under axial rotation. Like the previous Fig. , ref. point can be seen in this diagram. Ref. stresses in loading are reported 24.19 and 23.94MPa in numerical and analytical methods, respectively.

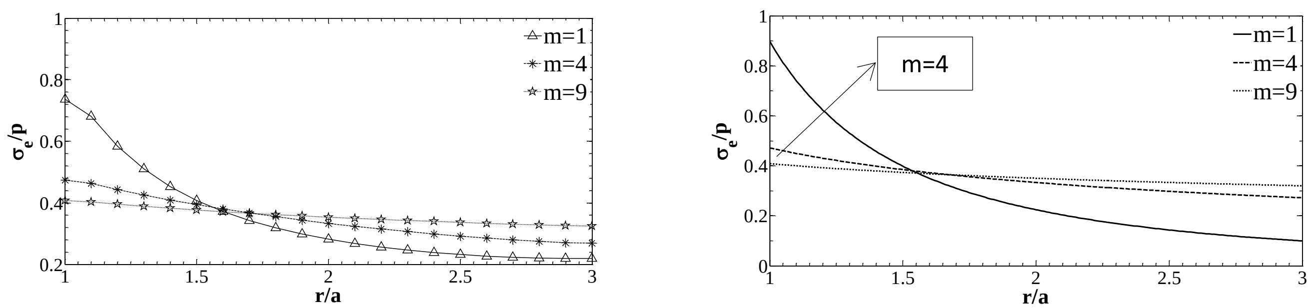
Fig. 4 shows effective dimensionless stress distribution in tank wall under combinational loading (internal pressure and axial rotation). The ref. stress in this type of loading is reported 76.9 and 75.4MPa in Finite Element and analytical methods, respectively.



(b) (a)

Fig. 2: effective dimensionless stress distribution in tank wall under 66-Mpa internal pressure

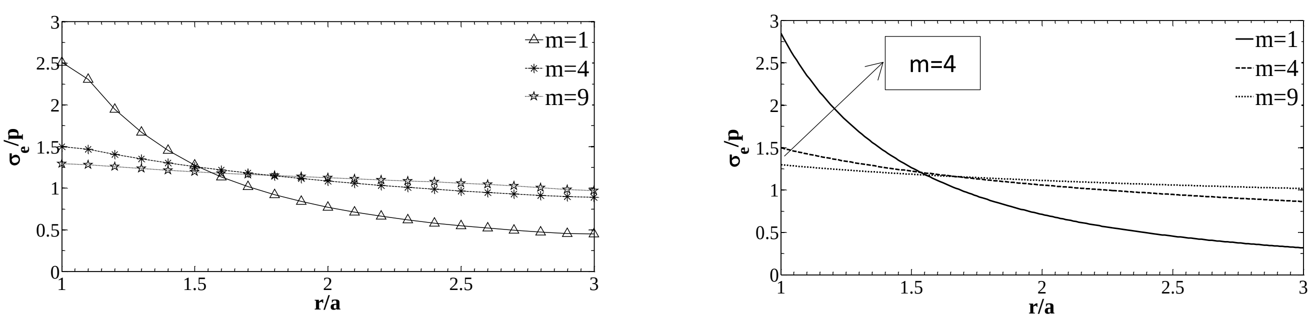
Analytical solution b- Numerical solution



(b) (a)

Fig. 3: effective dimensionless stress distribution in tank wall under axial rotation

a- Analytical solution b-Numerical solution



(b) (a)

Fig. 4: effective dimensionless stress distribution in tank wall under combinational rotation

a- Analytical solution b-Numerical solution

Ref. point in analytical solution is equally reported for all three loading modes at $r/a = 1.57$ In internal loading mode, the ref. point is $r/a = 1.62$ in Finite Element method. The difference lies in the fact that ref. point was obtained from intersection of elastic diagram and related diagram of elastic- completely plastic material in analytical method; however, ref. point in numerical solution was obtained from Norton equation for different values of power, so it is more realistic.

Table 2 shows ref. stress for all three loading modes with numerical and analytical methods. According to the results, ref. stress is obtained from numerical solution using equation (30) with 1.25 error. Ref. stress is generally the total sum of ref. stresses obtained from internal pressure and axial rotation separately. Since determining the coefficient for basic equations is difficult in creep studies and experimental study of creep behavior of structures is costly, this error is acceptable. Thus, the accuracy of proposed equation is approved to estimate ref. stress with α_i and β_i coefficients.

Table 2: Comparison of ref. stress (MPa) in three loading modes

Loading	Analytical	Numerical	Proposed equation	Error percentage
$P = 66\,[MPa]$ $\omega = 0$	52	56.1	-	7.88
$P = 0$ $\omega = 312\,[rad/sec]$	23.94	24.19	-	1.04
$P = 66\,[MPa]$ $\omega = 312\,[rad/sec]$	-	76.9	75.94	1.26

Fig. 5 and 6 show radius and perimeter dimensionless stress distribution along the tank wall in three various loading modes by numerical and analytical methods. It is obvious that an acceptable compatibility is seen between numerical data and the results of analytical method, showing the accuracy of Finite Element analysis.

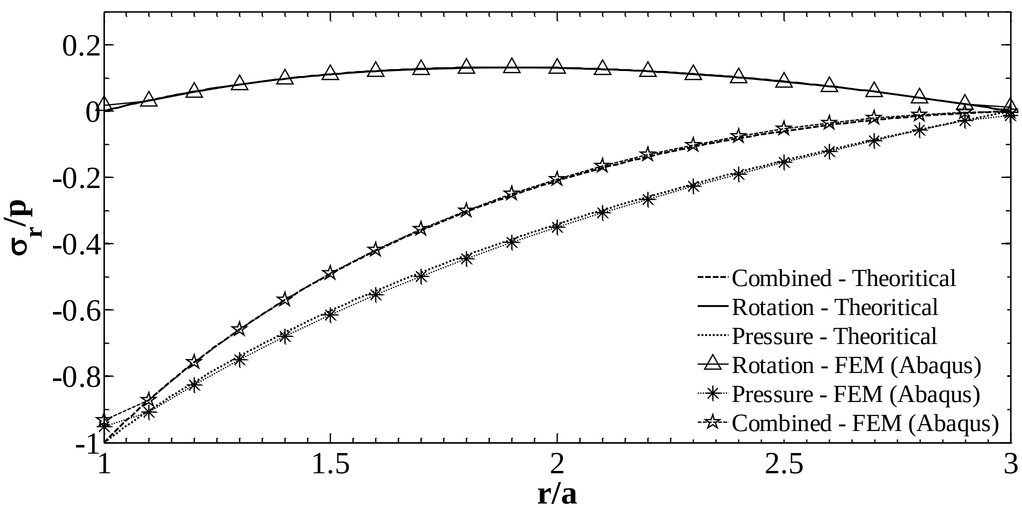


Fig. 5: Radial dimensionless stress distribution in tank wall in three loading modes by numerical and analytical method

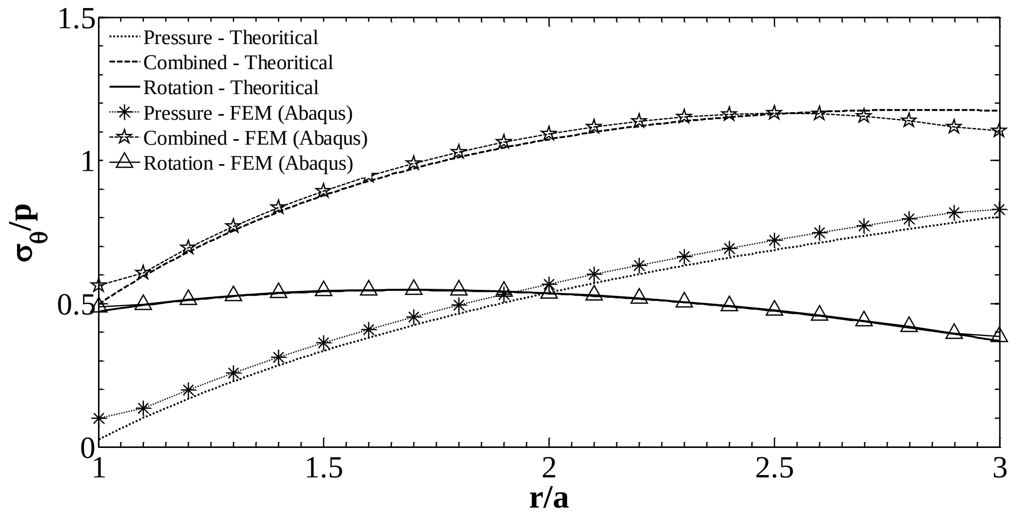


Fig. 6: perimeter dimensionless stress distribution in tank wall in three loading modes by numerical and analytical method

Equations 34 and 36 are the strength design constraints and creep lifetime for cylindrical tank thickness optimization. The intersection of strength safety factor diagram and constant creep lifetime with optimum thicknesses, some limitations are taken into account for design parameters. Fig. 7 shows such limitations for 2 and 3 static safety coefficient and 1000 and 10000-hour creep lifetime at 760 degrees Celsius.

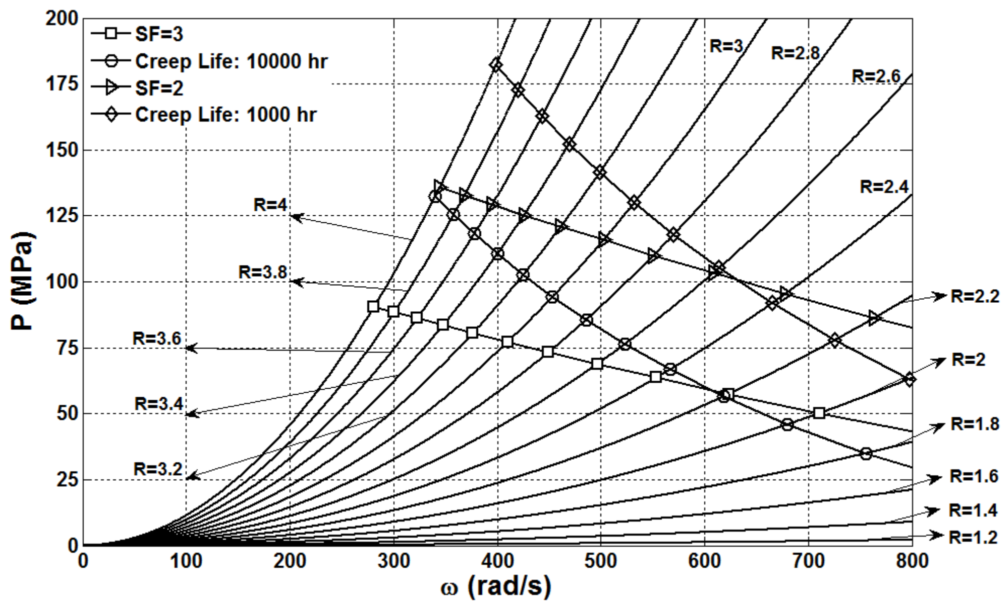


Fig. 7: The diagram to determine optimal radial ratio for cylindrical tanks under rotating pressure according to maximum creep lifetime and strength constraints. The material of alloy is INCONEL 718 at 760 degrees Celsius

Taking maximum creep lifetime and appropriate safety coefficient constraint concerning strength into account, a diagram such as Fig. 7 can be obtained by solving the equations and physical characteristics of the material in terms of yield stress and creep lifetime. This diagram is drawn for tank under rotating pressure made up of INCONEL 718 at 760 degrees Celsius. The intersection of design safety coefficient diagram and constant creep lifetime diagram with optimal radius ratio diagram, optimum tank specifications can be determined. Generally, according to Fig. 7, one of design criteria (static and creep) will be important. It is concluded that if rotating speed exceeds a certain limit, depending on creep lifetime (or internal pressure goes under a certain limit), then creep criterion will be the decisive factor in designing. If pressure exceeds a certain limit and rotating speed is less than a certain limit, then static design will be prominent. This pressure of rotating speed can be named as transient design mode. In other words, optimum thickness, maximum creep lifetime, and strength safety coefficient can be forecasted through the diagram in Fig. 7 for each certain pressure and working rotation speed.

The criterion to use ref. stress in creep lifetime prediction is one of approximately but widely-used issues in industries. According to Fig. 4, maximum stress occurs in internal tank wall which is greater than ref. stress. It is noteworthy that choosing the maximum stress as design criterion creates creep lifetime dependency to material (material creep coefficient). The constant creep value ranges between 3 and 9 for rotating pressure vessels used in industries (1980). Table 3 lists maximum stress to ref. stress ratio for different *m*s.

Table 3: maximum to ref. stress ratio for various creep constants

m(Creep constant)	$\sigma_{\max}/\sigma_R$
3	1.23
4	1.16
9	1.04

The results were taken from Fig. 4. It is approximately stated that maximum stress and ref. stress have linear relationship through $1/m$ parameter:

$$\frac{\sigma_{\max}}{\sigma_R} = \frac{0.84}{m} + 0.95 \qquad (۳۷)$$

Ref. stress can be approximately considered as criterion as various m values. However, the effect of maximum stress can be considered through equation (37) for higher level of safety in creep lifetime design.

6. CONCLUSION

Pressure vessels working at high temperature with axial rotation are widely used in advanced industries in particular in gas and oil as well power industries. For this reason, studying the creep behavior seems necessary for correct design of tanks. In this paper, we proposed a method to determine ref. stress for tanks which are under combinational loading. The comparison of results between data obtained through Finite Element Method (numerical solving) confirms the accuracy of proposed equation. This equation can be easily used to determine ref. stress in tanks with combinational loads. We are also able to predict structure behavior by single axial creep test with the mentioned stress. According to this, design diagram is presented for a tank made up of INCONEL 718. Despite the importance of this diagram, it is noteworthy that preparing it does not require analytical and numerical study of tank creep behavior and it is prepared with simple equations.

In terms of tank with mentioned material, the results show that rotating speed rise is the main design factor; however, strength constraint or safety factor is the main agent in low rotating speeds. In addition, it is seen that strength constraints loses its importance for high creep lifetime.

7. References

1. H. Chen, Z. Cen, B. Xu, and S. Zhan, A numerical method for reference stress in the evaluation of structure integrity, International journal of pressure vessels and piping, 1997,vol. 71, pp. 47-53.

2. H. Jahed and J. Bidabadi, An axisymmetric method of creep analysis for primary and secondary creep, International journal of pressure vessels and piping, vol. 80, 2003, pp. 597-606.

3. I. W. Goodall, R. A. Ainsworth, Assessment procedure for the high temperature response of structures, International Union of Theoretical and Applied Mechanics, 1991, pp. 303-311

4. J. F. dos Reis Sobrinhoa and L. de Oliveira Buenob, "Correlation Between Creep and Hot Tensile Behaviour for 2.25 Cr-1Mo Steel from 500ºC to 700ºC. Part 2: An Assessment According to Different Parameterization Methodologies," Revista Matéria, vol. 10, pp. 463-471, 2005.

5. H. Kraus, "Creep analysis," Research supported by the Welding Research Council. New York, Wiley-Interscience, 1980. 263 p, 1980.

6. N. Bhatnagar, P. Kulkarni, and V. Arya, Creep analysis of an internally pressurised orthotropic rotating cylinder, Nuclear engineering and design, 1984, vol. 83, pp. 379-388.

7. N. Bhatnagar, P. Kulkarni, and V. Arya, Creep analysis of orthotropic rotating cylinders considering finite strains, International journal of non-linear mechanics, 1986,vol. 21, pp. 61-71.

8. J. Jelwan, M. Chowdhury, and G. Pearce, Creep life design criterion and its applications to pressure vessel codes, Materials Physics and Mechanics, 2011,vol. 11, pp. 157-182.

9. R. K. Penny and D. L. Marriott, Design for creep, Springer Science & Business Media, 1995.

10. M. Davoudi Kashkoli and M. Zamani Nejad, Effect of Heat Flux on Creep Stresses of Thick-Walled Cylindrical Pressure Vessels, Journal of Applied Research and Technology, 2014,vol. 12, pp. 585-597.

11. T. Pankaj, Elastic-plastic transition stresses in rotating cylinder by finite deformation under steady-state temperature, Thermal Science, 2011,vol. 15, p. 537.

12. Z. Hoseini, M. Zamani Nejad, A. Niknejad, and M. Ghannad, New exact solution for creep behavior of rotating thick-walled cylinders, Journal of Basic and Applied Scientific Research, 2011,vol. 1, pp. 1704-1708.

13. NRIM Creep Data Sheet, No.3B, (1986).

14. R. Sim and R. Penny, Plane strain creep behaviour of thick-walled cylinders, International Journal of Mechanical Sciences, 1971,vol. 13, pp. 987-1009.

15. H. Altenbach, Y. Gorash, and K. Naumenko, Steady-state creep of a pressurized thick cylinder in both the linear and the power law ranges, Acta Mechanica, vol. 195, 2008, pp. 263-274.

16. T. Pankaj, Steady thermal stress and strain rates in a rotating circular cylinder under steady state temperature, Thermal Science, 2014,vol. 18, pp. 93-106.

17. T. Pankaj, Stresses in a thick-walled circular cylinder having Pressure by using concept of generalized strain measure, Kragujevac Journal of Science, 2013,vol. 35, pp. 41-48.

18. S. Sharma, Thermo creep transition in non-homogeneous thick-walled rotating cylinders, Defense Science Journal, 2009, vol. 59, pp. 30-36.

19. M. Zamani Nejad and M. Davoudi Kashkoli, Time-dependent thermo-creep analysis of rotating FGM thick-walled cylindrical pressure vessels under heat flux, International Journal of Engineering Science, 2014, vol. 82, pp. 222-237.